

# CS221: Logic Design

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# Digital Fundamentals

## CHAPTER Counters

# Counters

## Counting in Binary

As you know, the binary count sequence follows a familiar pattern of 0's and 1's.

The next bit changes on every fourth number.

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

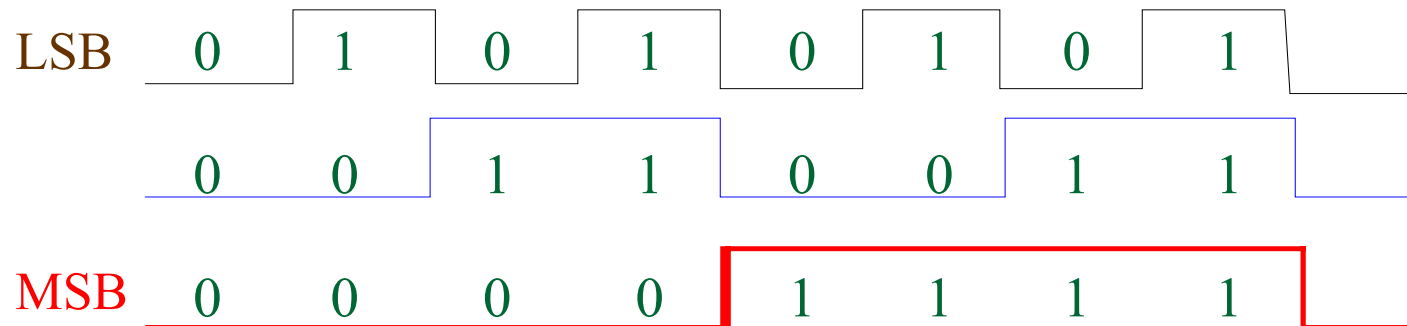
LSB changes on every number.

The next bit changes on every other number.

# Counters

## Counting in Binary

A counter can form the same pattern of 0's and 1's with logic levels. **The first stage in the counter represents the least significant bit** – notice that these waveforms follow the same pattern as counting in binary.



# Asynchronous binary counter

In an asynchronous counter, **the clock is applied only to the first stage.**

Subsequent stages derive the clock from the previous stage.

It uses J-K flip-flops in the **toggle mode.**

Notice that the Q0 output is triggered on the **leading edge of the clock signal.** The following stage is triggered from **Q0.**

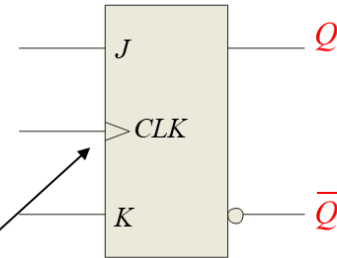
# Edge-Triggered Flip-Flops

## Edge-triggered J-K flip-flop

Inputs			Outputs		Comments
J	K	CLK	Q	$\bar{Q}$	
0	0	↑	$Q_0$	$\bar{Q}_0$	No change
0	1	↑	0	1	RESET
1	0	↑	1	0	SET
1	1	↑	$\bar{Q}_0$	$Q_0$	Toggle

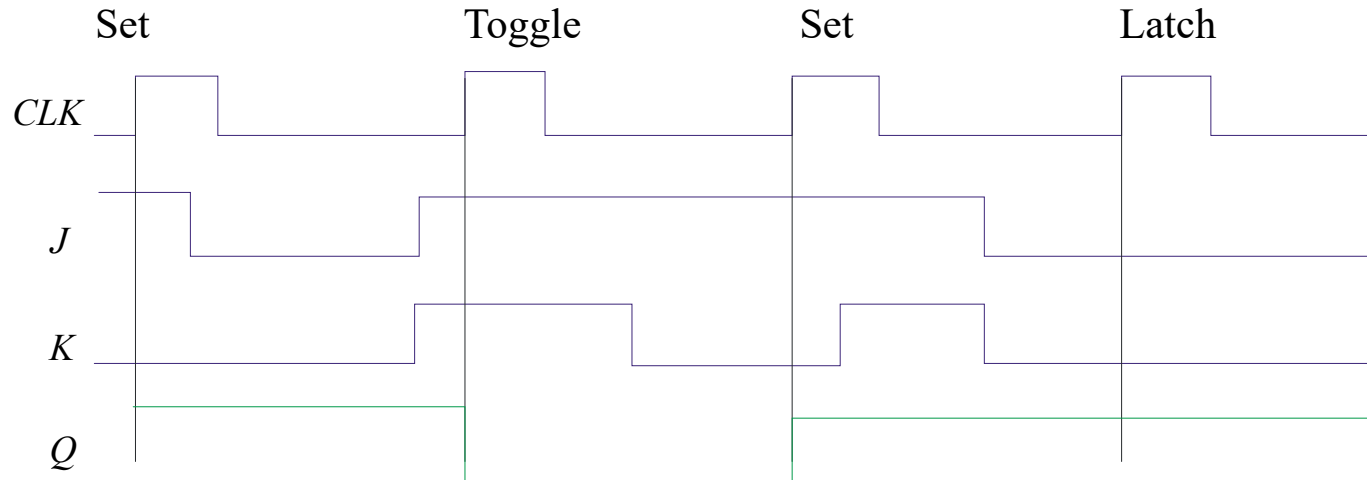
## Example

Determine the  $Q$  output for the  $J$ - $K$  flip-flop, given the inputs shown.



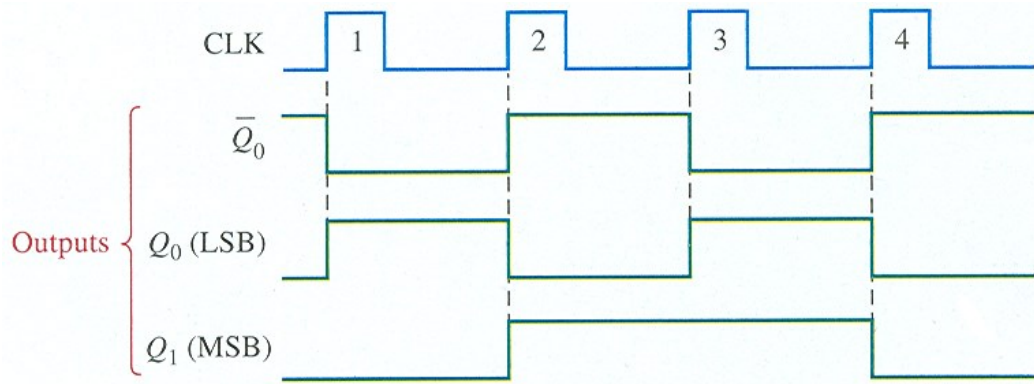
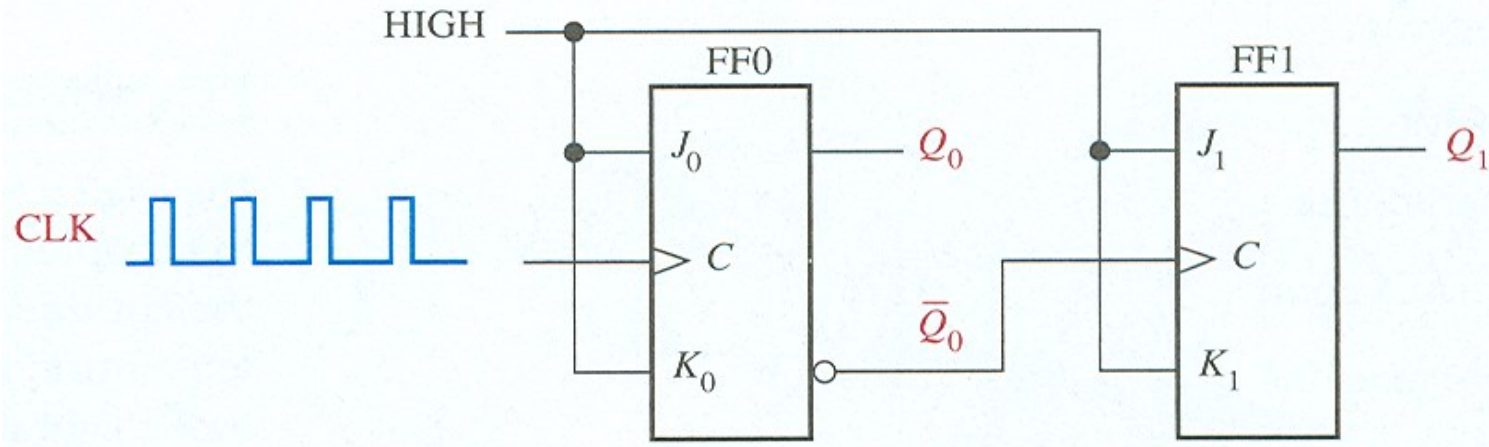
Notice that the outputs change on the leading edge of the clock.

## Solution



# Asynchronous binary counter

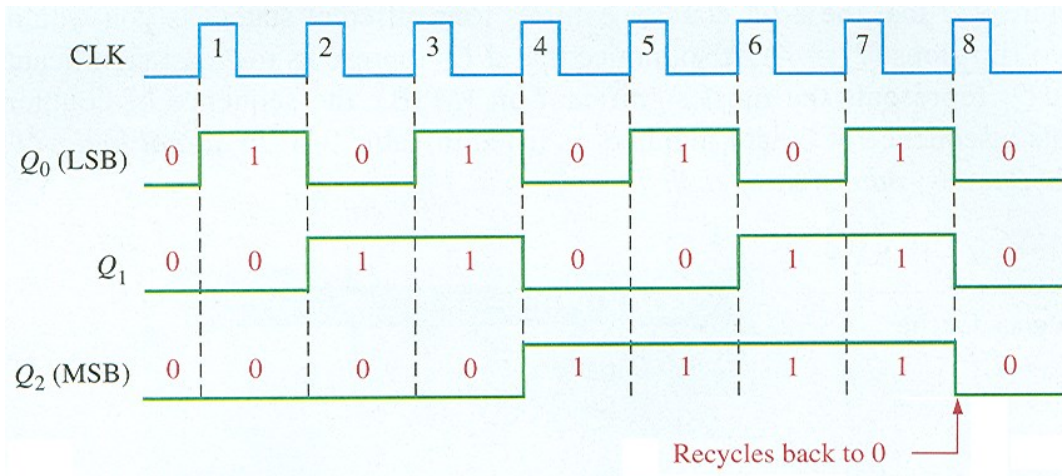
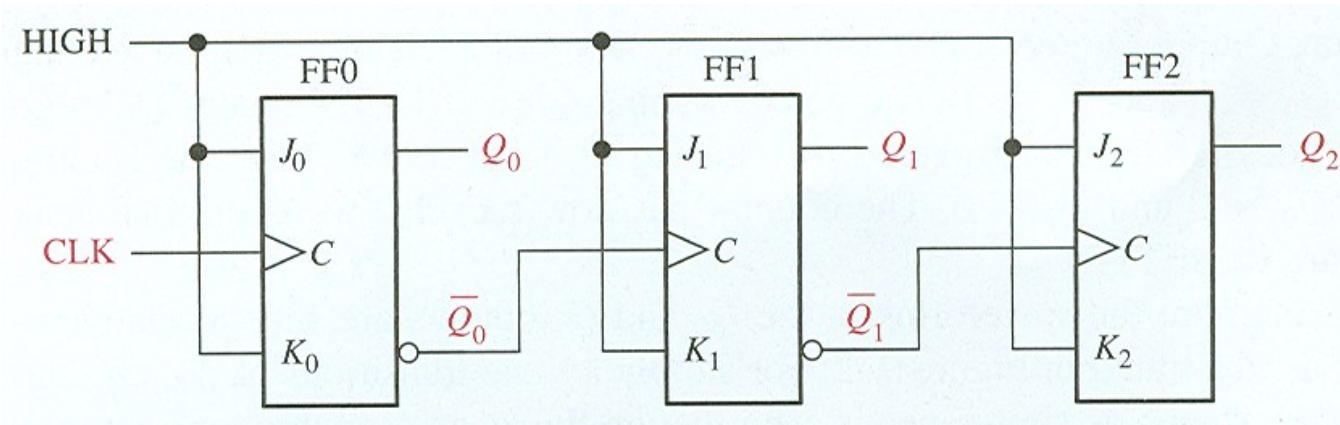
## 2-bit Asynchronous binary counter



CLOCK PULSE	$Q_1$	$Q_0$
Initially	0	0
1	0	1
2	1	0
3	1	1
4 (recycles)	0	0

# Asynchronous binary counter

## 3-bit Asynchronous binary counter

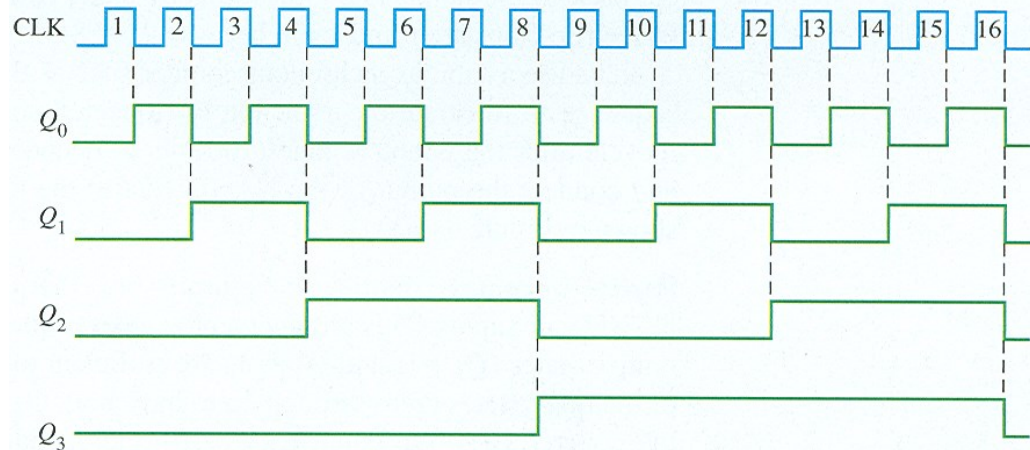
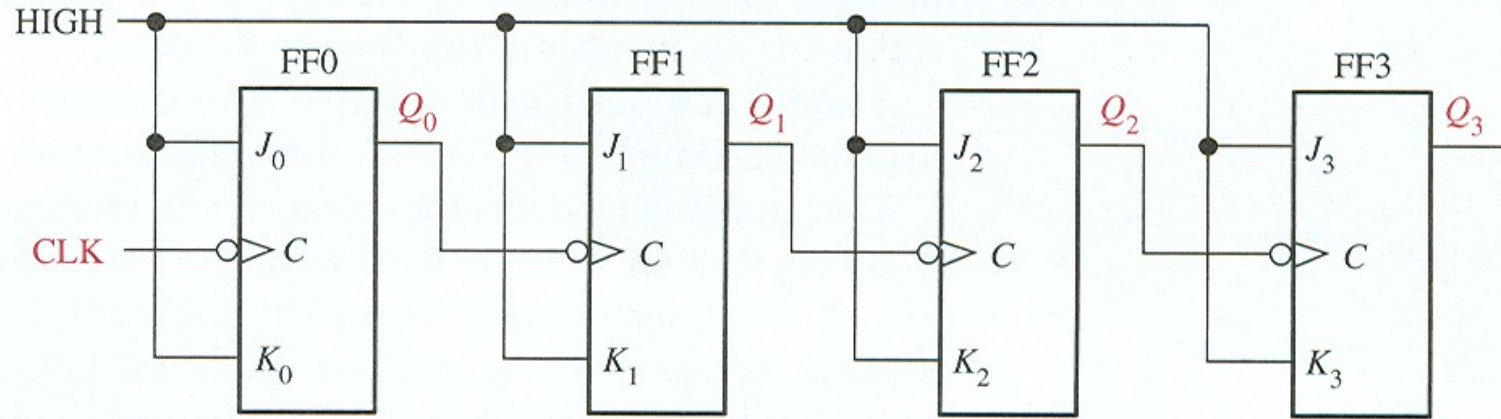


CLOCK PULSE	Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>
Initially	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8 (recycles)	0	0	0



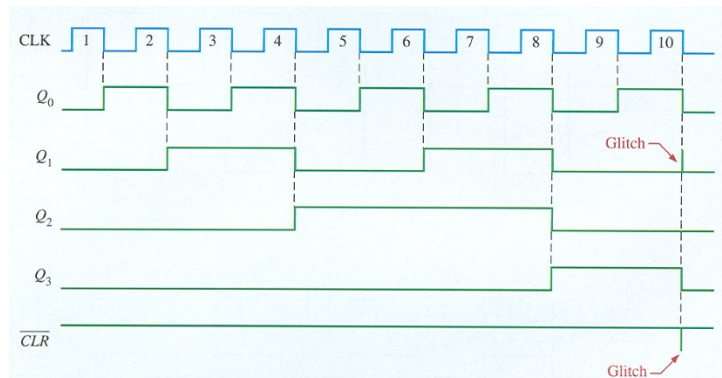
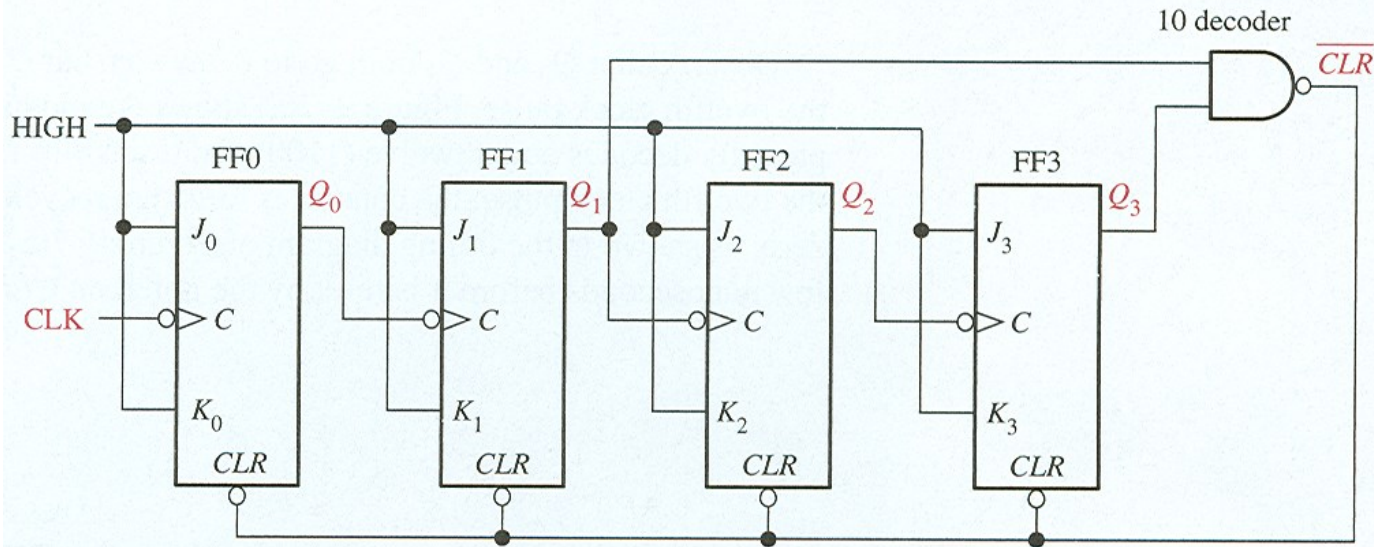
# Asynchronous binary counter

## 4-bit Asynchronous binary counter



# Asynchronous binary counter

## Asynchronous decade counter

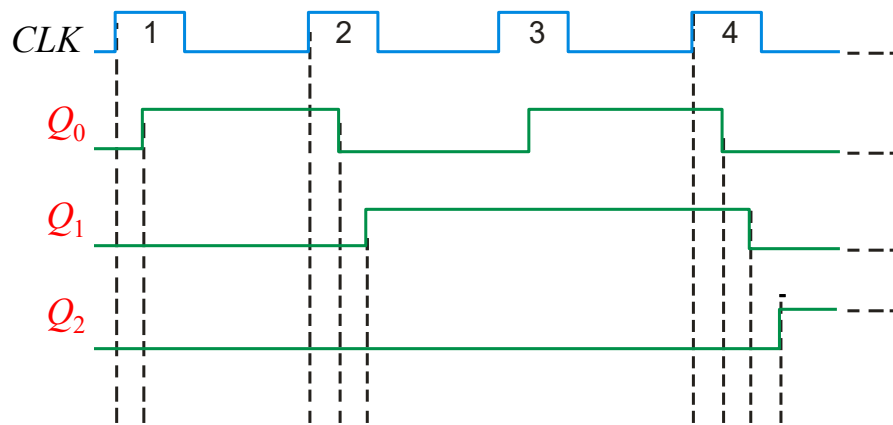


# Asynchronous binary counter

## Propagation Delay

Asynchronous counters are sometimes called **ripple counters**, because the stages do not all change together. For certain applications requiring high clock rates, this is a major disadvantage.

Notice how delays are cumulative as each stage in a counter is clocked later than the previous stage.



$Q_0$  is delayed by 1 propagation delay,  $Q_2$  by 2 delays and  $Q_3$  by 3 delays.

# Synchronous binary counter

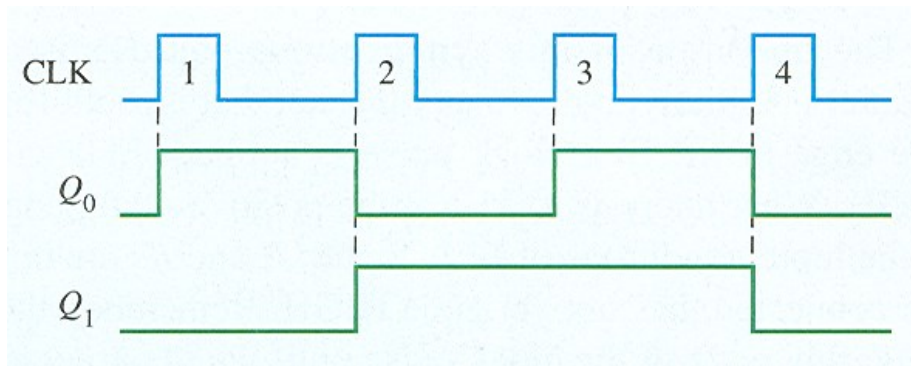
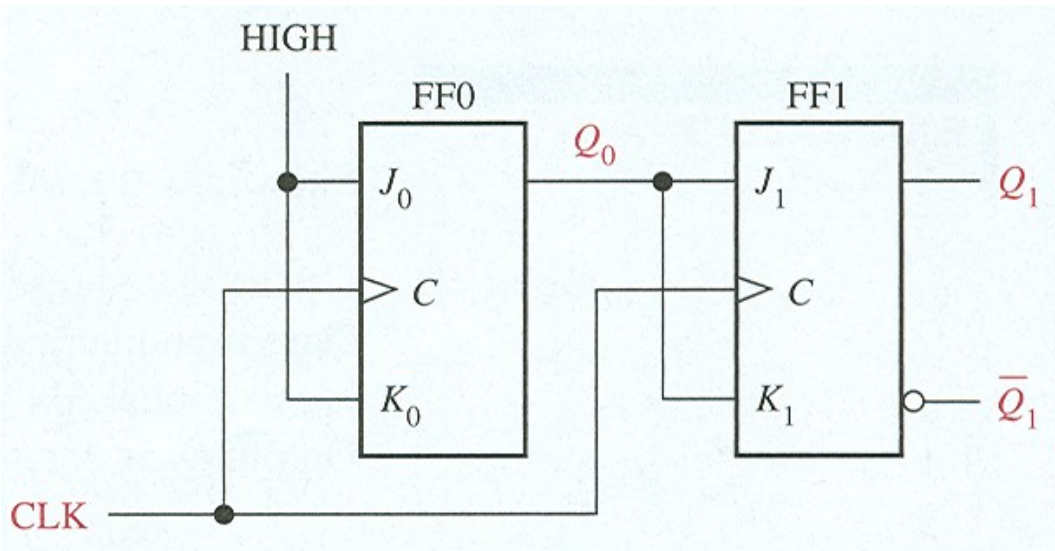
## Synchronous Counters

In a **synchronous counter** all flip-flops are **clocked together with a common clock pulse**.

Synchronous counters overcome the disadvantage of accumulated propagation delays, but generally they **require more circuitry to control states changes**.

# Synchronous binary counter

## 2-bit Synchronous binary counter

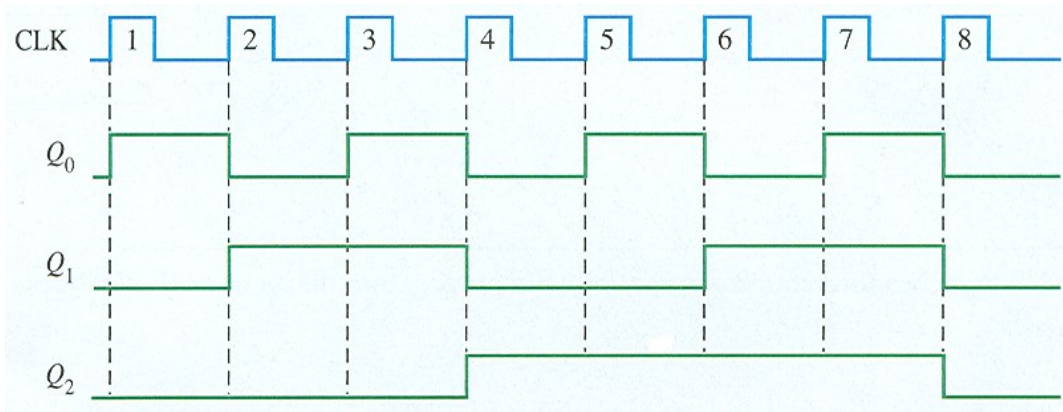
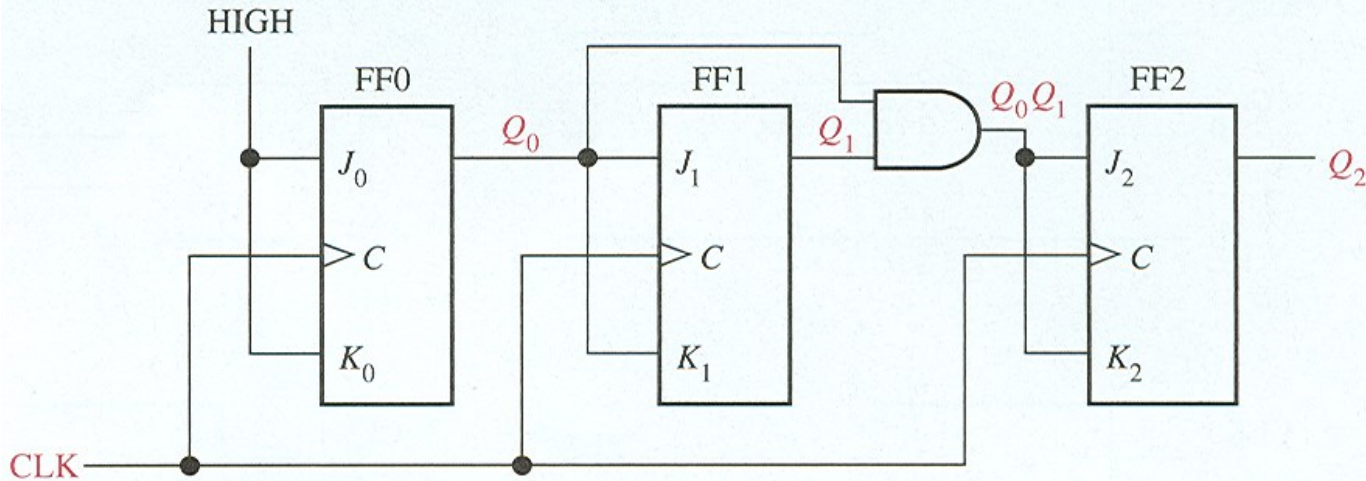


Inputs			Outputs		Comments
J	K	CLK	Q	$\bar{Q}$	
0	0	↑	Q <sub>0</sub>	$\bar{Q}_0$	No change
0	1	↑	0	1	RESET
1	0	↑	1	0	SET
1	1	↑	$\bar{Q}_0$	Q <sub>0</sub>	Toggle



# Synchronous binary counter

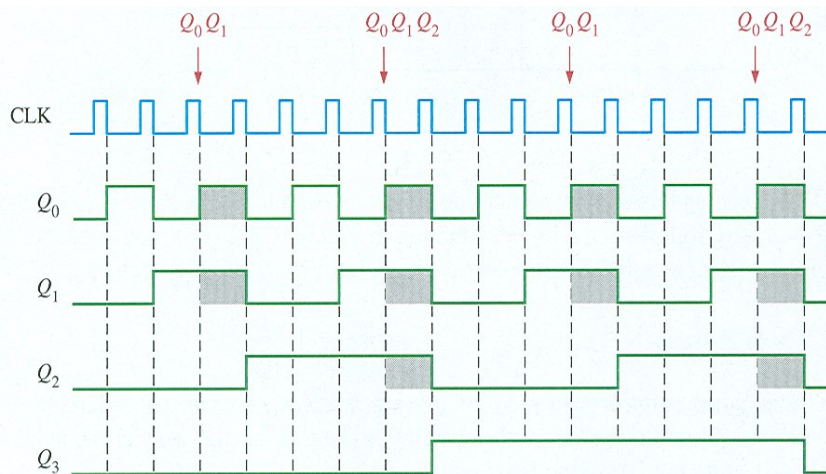
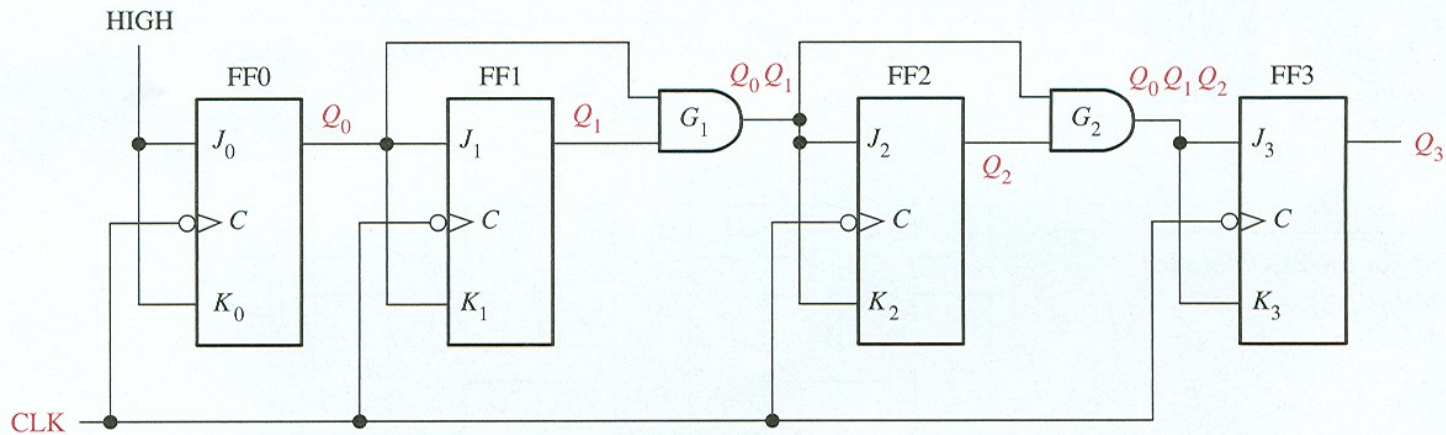
## 3-bit Synchronous binary counter



CLOCK PULSE	$Q_2$	$Q_1$	$Q_0$
Initially	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8 (recycles)	0	0	0

# Synchronous binary counter

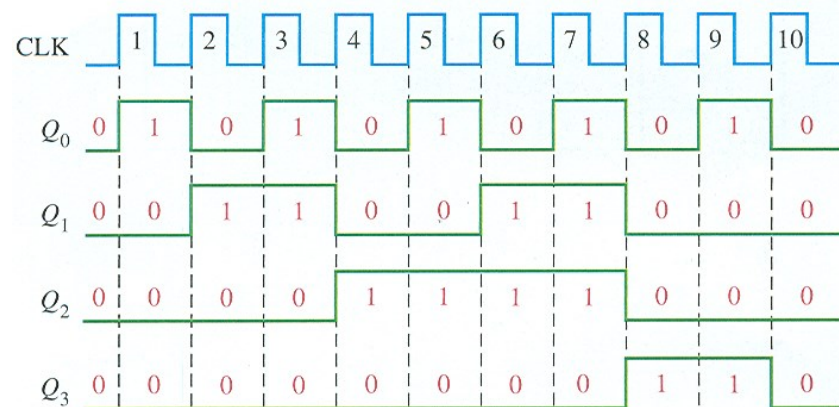
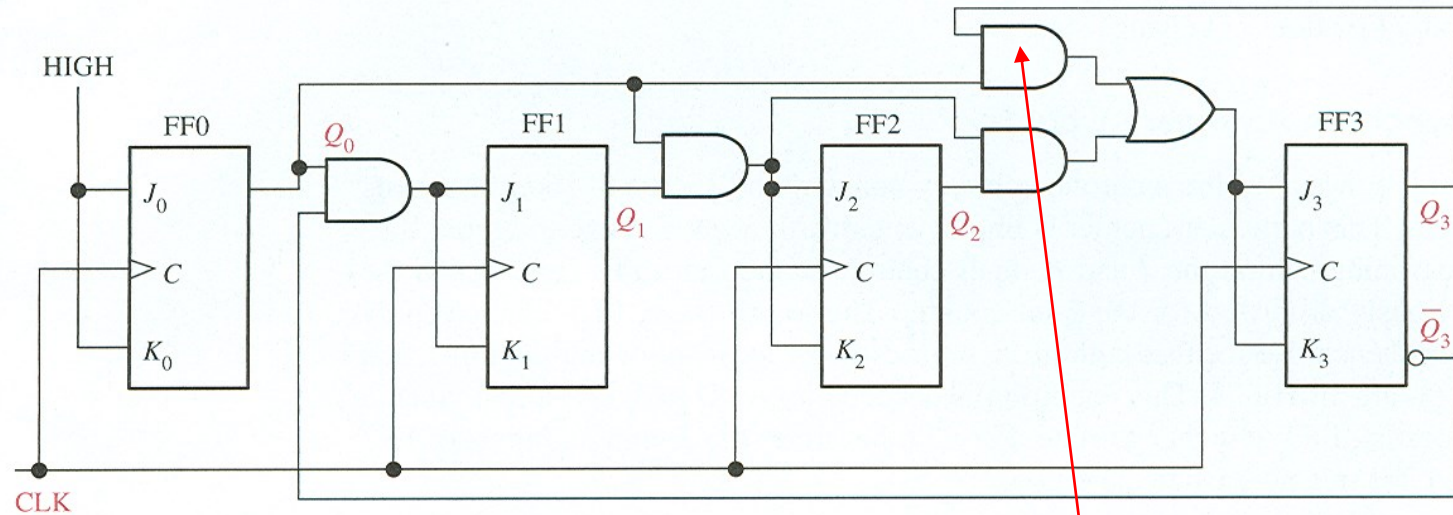
## 4-bit Synchronous binary counter



The 4-bit binary counter has one more AND gate than the 3-bit counter just described. The shaded areas show where the AND gate outputs are HIGH causing the next FF to toggle.

# Synchronous binary counter

## Synchronous decade counter



This gate detects 1001, and causes FF3 to toggle on the next clock pulse. FF0 toggles on every clock pulse. Thus, the count starts over at 0000.

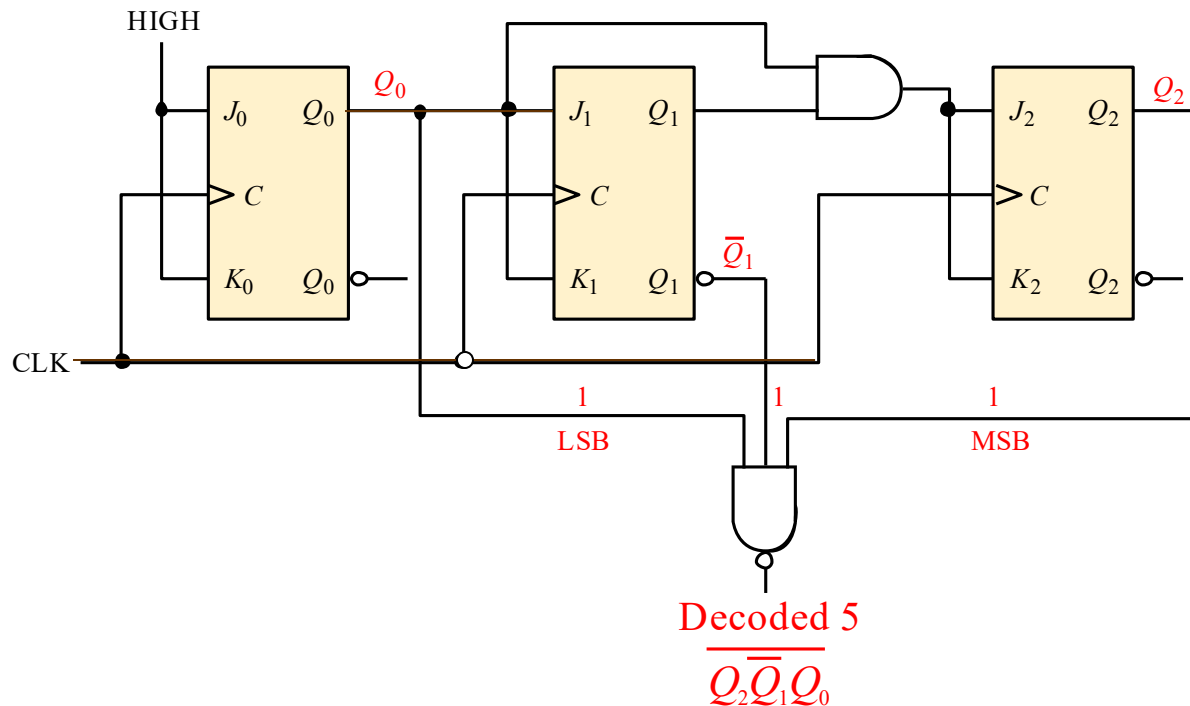


# Synchronous binary counter

## Counter Decoding

### Example

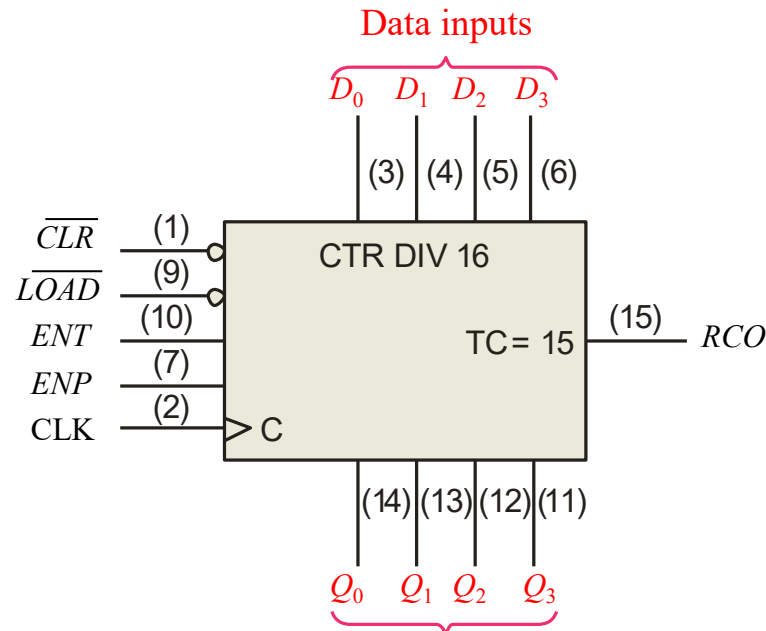
Show how to decode state 5 with an active LOW output.



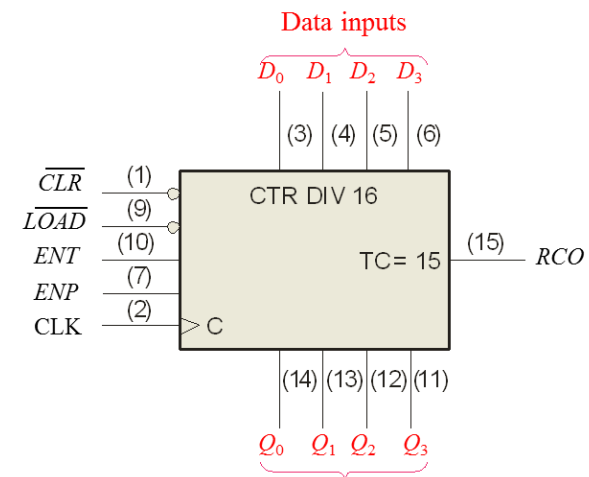
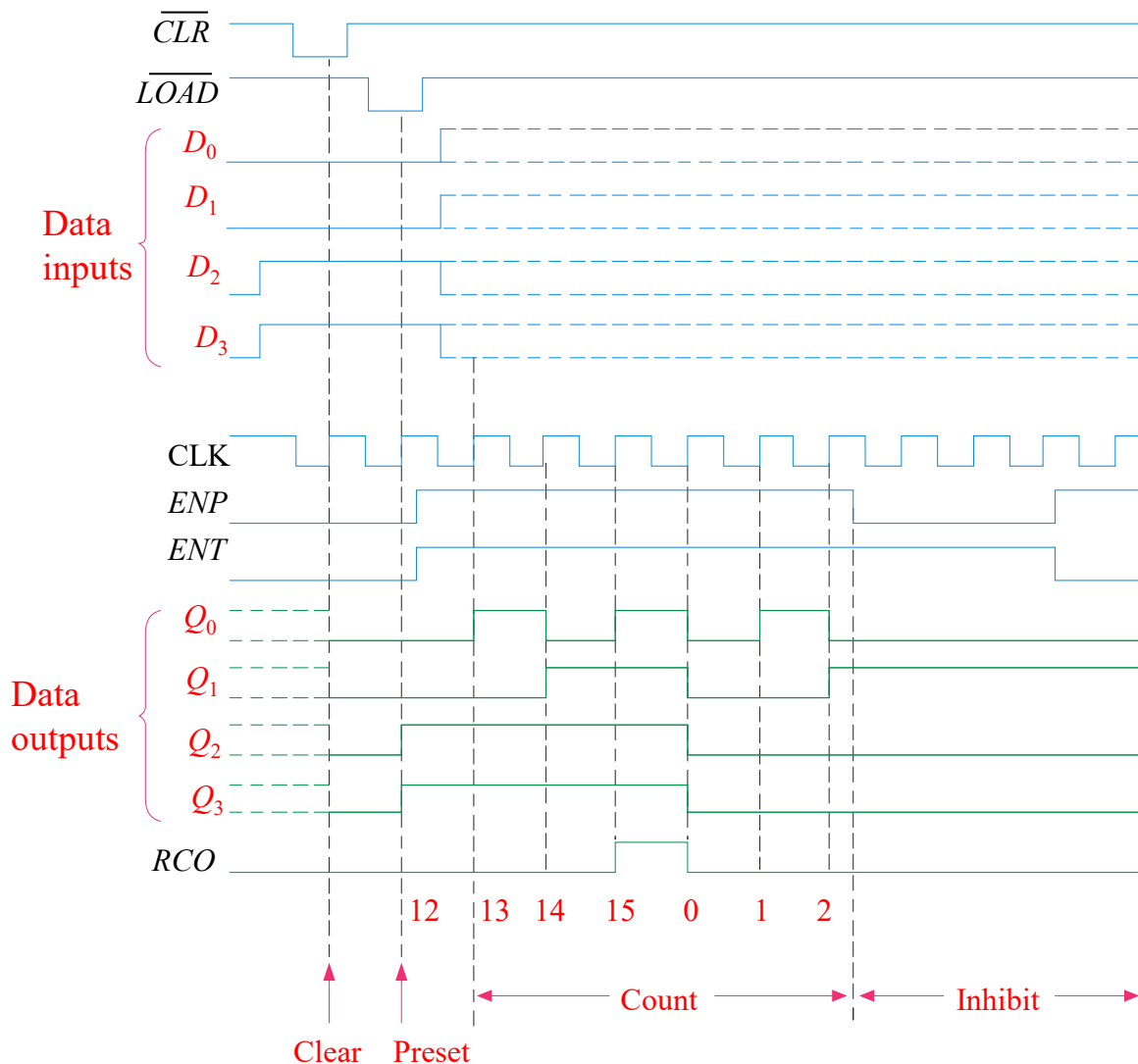
# Synchronous binary counter

## A 4-bit Synchronous Binary Counter

The **74LS163** is a **4-bit IC synchronous counter** with additional features over a basic counter. It has parallel load, a  $\overline{CLR}$  input, two chip enables, and a ripple count output that signals when the count has reached the terminal count.



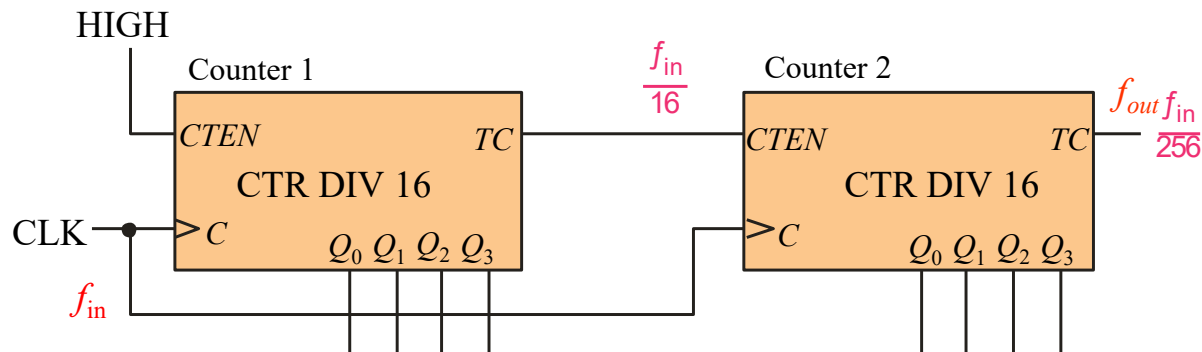
# A 4-bit Synchronous Binary Counter



# Synchronous binary counter

## Cascaded counters

Cascading is a method of achieving higher-modulus counters. For synchronous IC counters, the next counter is enabled only when the terminal count of the previous stage is reached.



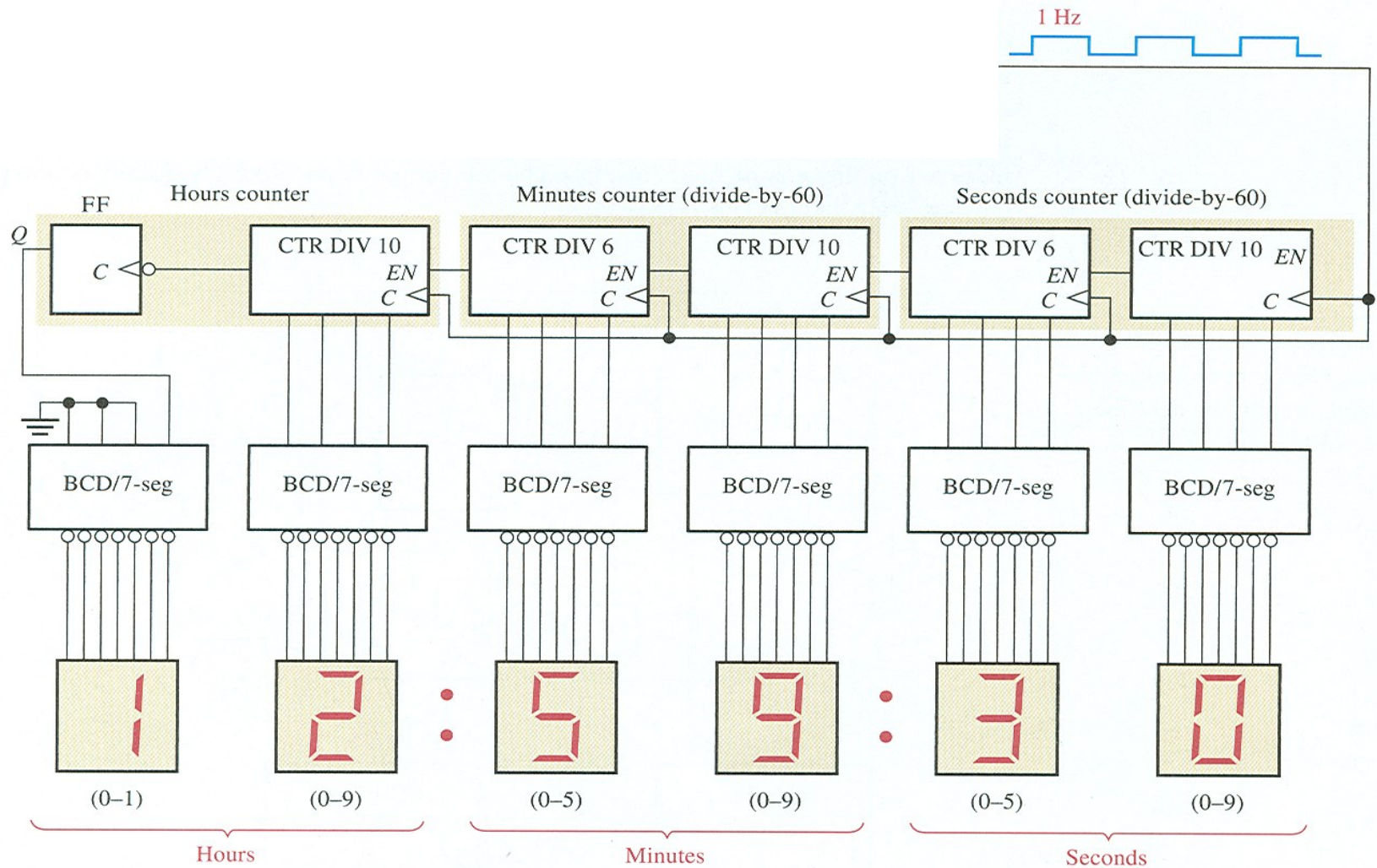
## Example

- What is the modulus of the cascaded DIV 16 counters?
- If  $f_{in} = 100$  kHz, what is  $f_{out}$ ?

## Solution

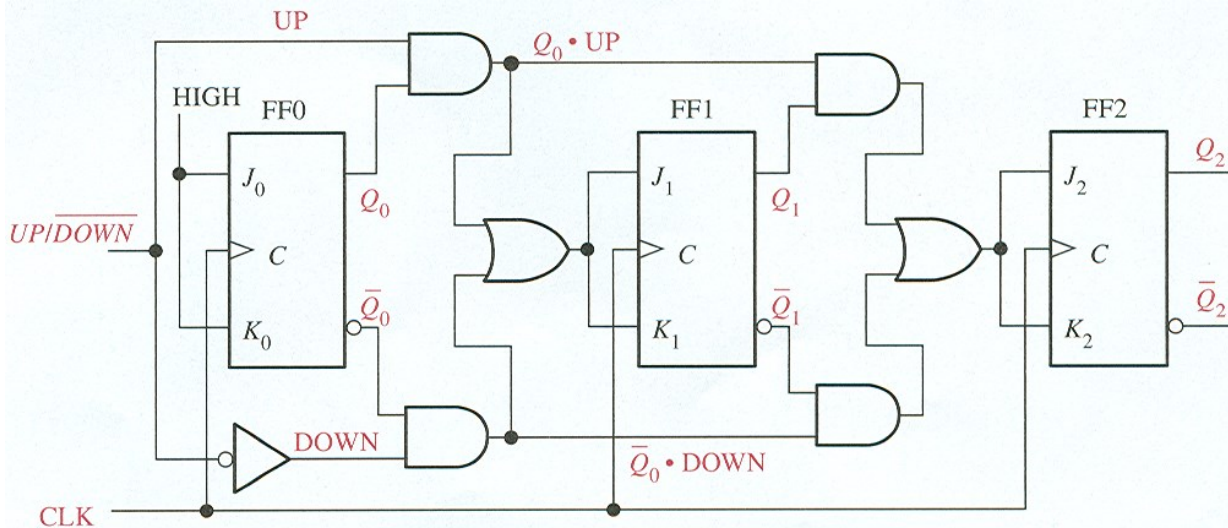
- Each counter divides the frequency by 16. Thus the modulus is  $16^2 = 256$ .
- The output frequency is  $100 \text{ kHz} / 256 = 391 \text{ Hz}$

# Counter Applications – Digital Clock



# Synchronous binary counter

## Up/Down Synchronous Counters



Up counter

Q1 Q0

0 0

0 1

1 0

1 1

Down counter

Q1 Q0  $\overline{Q_0}$

0 0 1

1 1 0

1 0 1

0 1 0

An up/down counter is capable of progressing in either direction depending on a control input.

CLOCK PULSE	UP	Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>	DOWN
0		0	0	0	
1		0	0	1	
2		0	1	0	
3		0	1	1	
4		1	0	0	
5		1	0	1	
6		1	1	0	
7		1	1	1	

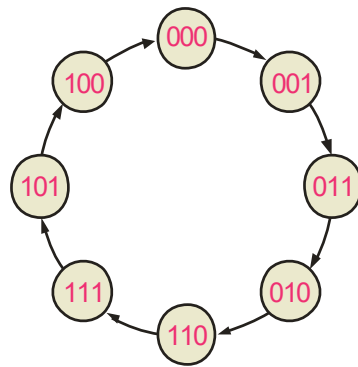
# Synchronous binary counter

## Synchronous Counter Design

Most requirements for synchronous counters can be met with available ICs. In cases where a special sequence is needed, you can apply a step-by-step design process.

**Start with the desired sequence and draw a state diagram and next-state table. The gray code sequence from the text is illustrated:**

State diagram:



Next state table:

Present State			Next State		
Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>	Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>
0	0	0	0	0	1
0	0	1	0	1	1
0	1	1	0	1	0
0	1	0	1	1	0
1	1	0	1	1	1
1	1	1	1	0	1
1	0	1	1	0	0
1	0	0	0	0	0



# Synchronous binary counter

## Synchronous Counter Design

The J-K transition table lists all combinations of present output ( $Q_N$ ) and next output ( $Q_{N+1}$ ) on the left. The inputs that produce that transition are listed on the right.

Output Transitions		Flip-Flop Inputs	
$Q_N$	$Q_{N+1}$	J	K
0	→ 0	0	X
0	→ 1	1	X
1	→ 0	X	1
1	→ 1	X	0

Each time a flip-flop is clocked, the  $J$  and  $K$  inputs required for that transition are mapped onto a K-map.

An example of the  $J_0$  map is:

	$Q_0$	0	1	
$Q_2Q_1$	00	1	X	$\bar{Q}_2\bar{Q}_1$
	01	0	X	
	11	1	X	$Q_2Q_1$
	10	0	X	

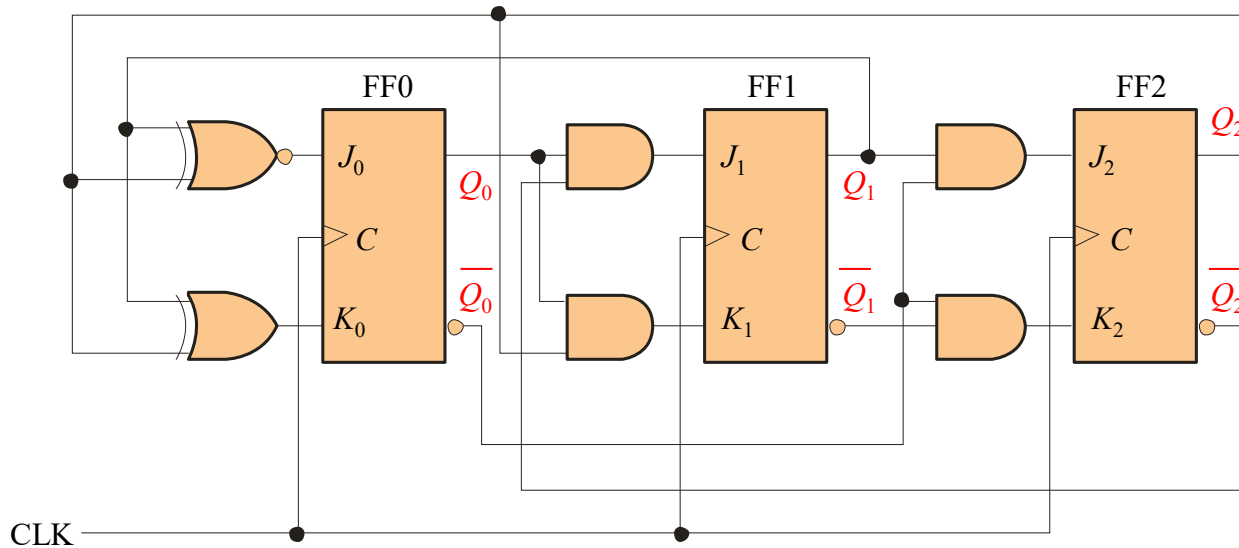
$J_0$  map

Present State			Next State		
$Q_2$	$Q_1$	$Q_0$	$Q_2$	$Q_1$	$Q_0$
0	0	0	0	0	1
0	0	1	0	1	1
0	1	1	0	1	0
0	1	0	1	1	0
1	1	0	1	1	1
1	1	1	1	0	1
1	0	1	1	0	0
1	0	0	0	0	0



# Synchronous binary counter

## Synchronous Counter Design



The logic for each input is read and the circuit is constructed.  
The slide shows the circuit for the gray code counter...